A Discussion of the Diagrams in Krugman Geography and Trade*, Chapter 1*

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Paul Krugman's *Geography and Trade* is rightly considered a cornerstone for modern urban economics (and for modern trade, maybe). Chapter 1 sets the stage by considering a simple theory of agglomeration based on the scale economies of the factory. The purpose of this note is to correctly draw the diagrams that describe the essence of that theory. The diagrams that appear in the book are misleading, and while not wholly incorrect, are difficult understand and in one instance, lead Krugman himself into error.

The theory goes as follows. There are two regions, East and West, and two industries, manufacturing and agriculture. Agricultural workers, of proportion $(1-\pi)$ are evenly split between East and West. The question is where should the manufacturing workers go, or more appropriately, where should the factories be? Just in the East, just in the West, or one factory in each region?

As in Krugman, let s_M be West's share of manufacturing employment, and s_N West's share of population. These are the two endogenous variables. West's share of population is determined by an accounting equation:

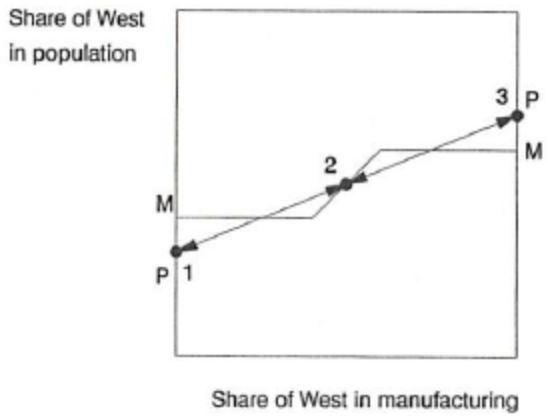
$$s_N = (1-\pi)/2 + \pi s_M \tag{1}$$

Half of the agricultural workers live in the West, and some manufacturing workers.

Manufacturing population is determined by the manufacturing firms' cost minimization. The tradeoff is this: factories have setup costs of F, and it costs t to ship a unit of output from one region to the other. So if there is one factory in the East you save F (from not building the second factory in the West), but pay txs_N in shipping costs, because x is total manufacturing output, of which s_N goes to the west. Similarly if you build one factory in the west, you save F, but pay $tx(1-s_N)$ in shipping cost. So s_M is given by:

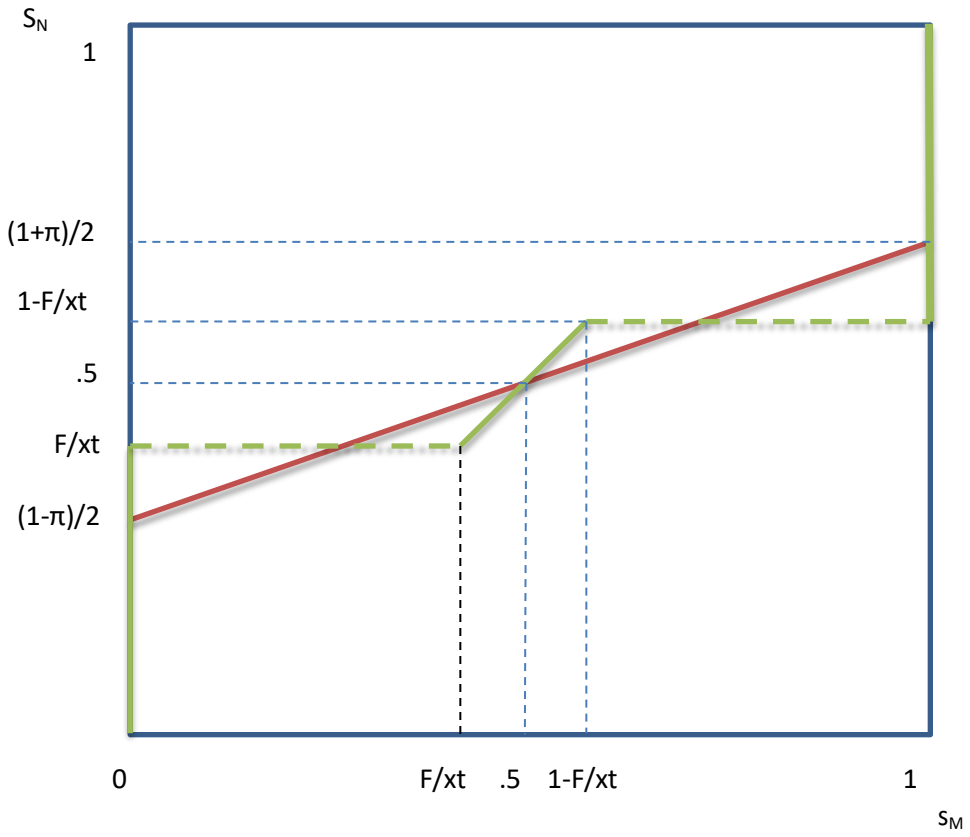
$$\begin{aligned} S_M &= 0 \text{ if } s_N < F/tx \\ S_M &= S_N \text{ if } F/tx < s_N < 1 - F/tx \\ S_M &= 1 \text{ if } s_N > 1 - F/tx \end{aligned} \tag{2}$$

Here is Krugman's representation of the model. PP is equation (1), a line with intercept $(1-\pi)/2$ and slope $\pi < 1$. MM is equation (2). It's already a little confusing because (2) appears to be a flat line segment (at zero), then a 45 degree line, then another flat segment (at unity). While that is how MM looks in the graph, this thinking has actually reversed the axes. Second, it looks for all the world like there are three equilibria, where the MM and PP intersect.



And Krugman *says* there are three equilibria. But they're not where those lines cross, they're where the points are labeled 1,2, and 3. The problem is that the horizontal segments of MM are not part of (2). Those values of s_M are undefined in (2). The flat segments in (2) are (obviously) *vertical* segments, but these vertical segments are not pictured in Krugman's figure. Whether through author carelessness or bad printing/proofing, they are overlaid onto the boundaries of the unit square.

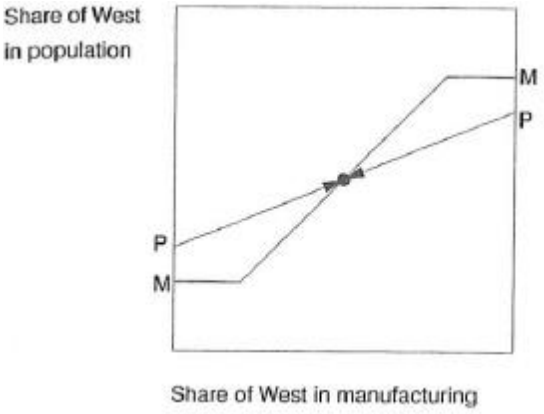
Here is what the diagram should look like, with correct intercepts and lines.



which makes it clear that the three equilibria are at 0, 0.5, and 1, and not at some equilibrium between 0 and 0.5 (or .5 and 1).

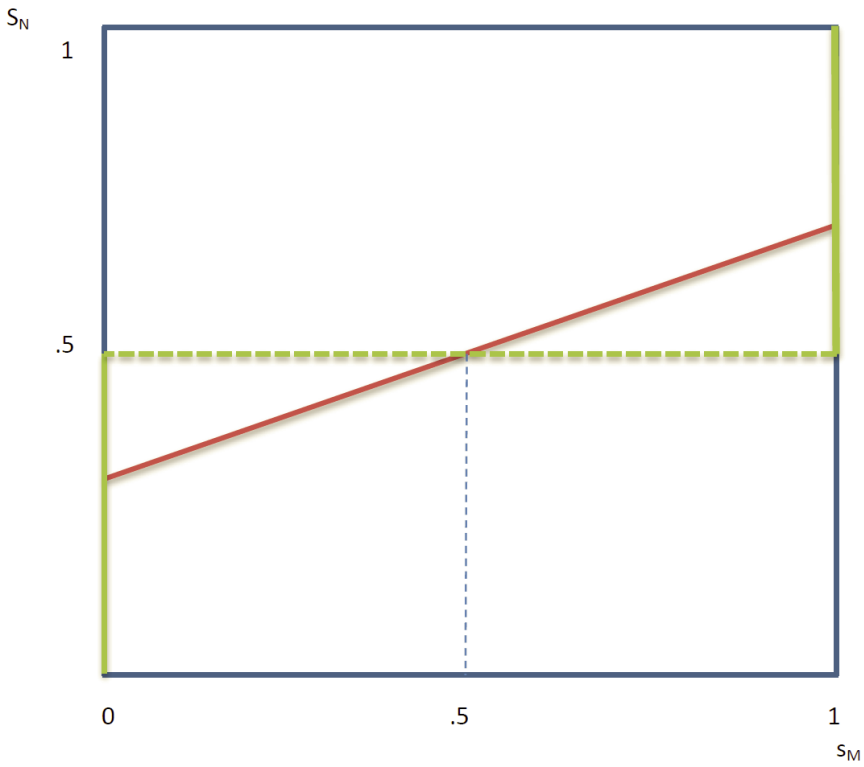
The above figure is for the case when $F/xt>(1-\pi)/2$—i.e. when fixed costs and manufacturing workforce are relatively high. It then behooves firm owner to locate in one place. Nevertheless the even split is also stable. See the text for the discussion of history vs. expectations in figuring out which of the three will hold.

The case where $F/xt<(1-\pi)/2$ is in Chapter 1's Figure 1.3.



I didn't redraw this case, but it's clear what happens. Again, the flat segments are not part of the defined function, but the segments of the unit square that proceed down (on the left) and up (on the right) are. But PP and MM only cross at the even split, and there is no agglomeration.

I mentioned that there was an actual error by Krugman. It comes in footnote 9, the important case where $F/tx>=.5$, which closes out the center section of the MM curve. For ease of exposition, let it equal .5. Krugman does not apply the appropriate diagram, but it would look like this:



Krugman's statement for this case is "MM is simply a horizontal line". As can be seen, this is an error. MM is everything *but* a horizontal line. If that statement were right, the equal split equilibrium would still hold. In fact, MM is two vertical lines, that correctly pinpoint the two equilibria at 0 and 1 (as Krugman goes on to say).